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Yintelligence™: The mapping of the pre-heaven or FuXi hexagrams to the post-heaven or King Wen hexagrams.

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Abstract

The Yijing[1] belongs to the famous group of the five classics. In the Ming and Qing dynasties, the *Four Books and Five Classics* were the subject of mandatory study by those Confucian scholars who wished to become government officials.

- The Book of Rites
- The Classic of Changes (Yijing, The Classic of Changes/Changes of the Zhou Dynasty!)
- The Classic of History (Book of Documents)
- The Classic of Poetry
- Spring and Autumn Annals

The Yijing consists of 64 figures called hexagrams[2] formed by six Yin (broken) or Yang (unbroken) lines[3] and commentaries on each of these hexagrams and their lines. The whole set of the hexagrams and their interdependencies make a dynamic model for the understanding or the anticipating of the different steps change goes through. The Yijing does not predict the future but gives a snap-shot of the actual situation and the holistic potentialities which may be deployed or not in order to better adapt to the on-going change. The different stages of a change are explained by the Yin-Yang polarity and the never-ending transformation of old-Yin in young-Yang, old-Yang, young-Yin and old-Yin again.
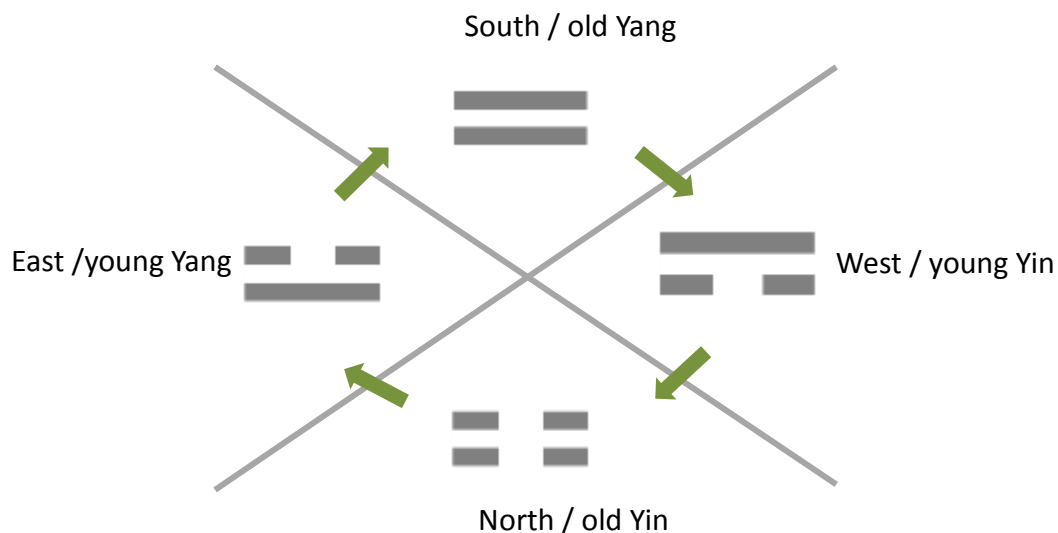
Figure 1: The 4-cycle of Yin-Yang Change and the 4 space region

The 64 hexagrams are built by pairing the 8 basic trigrams obtained by adding to each bigram a supplementary Yin or Yang line. Figure 1 depicts a 4-regions space (East, South, West, North) and a cyclic movement. A forthcoming article will explain the relation between this space organization and the whole set of 64 hexagrams[4] as a network. For now, the cyclic change is of major interest.

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[1] (Faure, et al., 2002); (R. Wilhelm, 1961); (Schlumberger, 1987)
[2] For a better understanding of the genesis of the hexagrams: (Javary, 2003), (Javary, 1997)
[3] (Ning Lu, 2008 S. page 23)
[4] Definition 4

Purpose

The 64 hexagrams of the Yijing are organized in 16 "first degree" nuclear families, which may be consolidated in 4 "second degree" nuclear families. There are another 16 families organizing the 64 hexagrams in a different structure. In the literature they are called the pre-heaven hexagrams[5] , in this paper they are referred to as the FuXi hexagrams. They form 16 cyclic families, whose structure will be analyzed in this paper.

As explained in 7.3, these cyclic families may express on the hexagram's level the changes due to the 4 different Yin and Yang phases. Thus they induce a "predecessor-successor" relationship between the hexagrams belonging to the same FuXi family.

$$H1 \longrightarrow H2 \longrightarrow H3 \longrightarrow H4 \longrightarrow H1$$
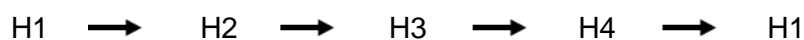
While the 4 members of each nuclear family are centered on a "master" hexagram that corresponds to a common inner lines' structure, each FuXi family is cyclically[6] structured and we call the hexagrams belonging to the same family FuXi[7] related.

Each Wen hexagram corresponds exactly to a FuXi hexagram. This correspondence reflects the respective trigrams' arrangements of King Wen and FuXi. The trigrams of the FuXi hexagram and those of the corresponding Wen hexagram occupy the same locations in the FuXi and the King Wen arrangements displayed in the Figure 6.

This paper explains the relation between the Wen hexagrams and their corresponding FuXi hexagrams based on structural and semantic properties.

Methodology

To each of the 32 pairs of opposite Wen hexagrams correspond 2 nuclear hexagrams, called FuXi nuclear pairs. The 2 hexagrams of a FuXi nuclear pair are specified by a first degree nuclear line structure and two opposite envelopes[8]. The resulting FuXi hexagrams related to the pair of Wen hexagrams are then a member of one of the nuclear families considered.

To map each of the 32 pairs of Wen hexagrams unambiguously to the right pair of FuXi hexagrams, two sets of particular hexagrams have been selected:
1. those obtained by doubling the 8 trigrams[9]
   1-The Creative, 2-The Receptive, 29-The Abysmal, 30-The Fire, 51-Thunder, 52-Moutain, 57-The Penetrating, 58-The Lake
2. those for which the opposite equals the reverse[10].
   11-Prosperity, 12-Adversity, 17-Following, 18-Decay, 53-Gradual Progress, 54-The Marrying Maiden, 63-After Completion, 64-Before Completion.

In addition 4 general principles to better determine the structure of the FuXi families have been formulated. The careful application of these 4 principles to the selected hexagrams allocates to each of the 8 basic Wen or post-heaven trigrams exactly one FuXi or pre-heaven trigram.

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[5] (Schlumberger, 1987 p. page 66)
[6] The 4-cyclic character of this relation is explained in section 0
[7] The post-heaven or explicate hexagrams are called Wen hexagrams and the pre-heave and n or implicate FuXi hexagrams.
[8] Definition 8. Therefore we also have 32 pairs of FuXi nuclear.
[9] Hexagrams names according to (R. Wilhelm, 1961)
[10] Definition 6 and Definition 7

Summary

It has been demonstrated, that the pre-heaven and post-heaven hexagrams are pairwise related through specific relations defined by the opposition of the post-heaven hexagrams and 2 members of particular nuclear families which have opposite envelopes. Basically, the links between pre-heaven or FuXi hexagrams and post-heaven or Wen hexagrams are based on the hexagrams' structure, especially on the opposition of 2 hexagrams. Two FuXi related hexagrams are connected to each other by a "predecessor-successor" like relation, the pre-heaven or FuXi hexagram somehow gives the necessary prerequisite which the post-heaven hexagram must own. In a pair of opposite hexagrams, the one possesses the complementary dimensions which are absent in the other. All these links may be represented using the trigrams deployed adequately on two circles, one with the pre-heaven trigrams, the FuXi arrangement, and the second with the post-heaven trigram or King Wen arrangement. Two corresponding trigrams are identically positioned in the two circular arrangements.

The logical and sophisticated system applied to link the Wen to the FuXi hexagrams again reveals how subtle the whole system of 64 hexagrams has been conceptualized and evidences a new and original insight into the structure of the Yijing.

Practical significance of the established result.

Usually a Yijing's casting consists of two principal hexagrams: a situation and a mutation if there are either old-Yin or old-Yang lines. To interpret the casting correctly it is mandatory to thoroughly understand what differentiates the significant situation and mutation hexagrams from the other 62 remaining hexagrams. To do this considering the nuclear, opposite, and the lines hexagrams[11], pre-heaven or FuXi, is very helpful. If the roles of the nuclear, opposite and line hexagrams are relatively known and clear, the importance of the pre-heaven or FuXi hexagram is generally not sufficiently recognized. The understanding of how the pre-heaven or FuXi hexagrams connect to the others will, no doubt, substantially strengthen the statements of a casting, and provide them with a better traceability and comprehension.

The whole set of post-heaven hexagrams is called the explicate casting. The FuXi hexagrams are used to enrich its interpretation by evidencing the implicate[12] casting related to the corresponding situation and mutation hexagrams. By linking the explicate hexagrams to their related FuXi hexagrams the interpretation of the situation unfolds and reveals correlations between the different parameters which influence the analyzed change process.
.It also contributes to substantially enforce
1. The specific understanding of the 64 hexagrams and their interdependencies, especially the symmetry existing between the hexagrams' geometry and the carried semantic elements.
2. The integration of the FuXi hexagrams in a casting's interpretation considerably helps to demonstrate the relevance and consistency of the casting by revealing hidden connections between the selected hexagrams.
3. The perception of the Yijing internal logic as a 4-valued logical system and therefore the support of a thin granular conceptual framework for decision making.
4. The assumption that the trigrams play a significant role in organizing the hexagrams especially to describe the mapping of the pre-heaven to the post-heaven hexagrams.

Zurich, Mai 2013
Prof. Dr. Gabriel Felley

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[11] Definition 9

[12] According to (Schöter, 2011) and his connection with the implicate and explicate order in quantum mechanics (Bohm, 1980). In the literature it is called the "hidden" casting but the naming "implicate casting" seems to be more appropriate and therefore the casting corresponding to the Wen hexagrams is called explicate.

Acknowledgement

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[13] The use of „entanglement„ is not fortuitous at all, it is more like a glance at the phenomena of entanglement experimentally verified in quantum mechanics. (Gisin, 2012)

Contents

Index of Illustrations and tables

Index of acronyms and definitions

Definition 1
Yin and Yang are considered as two basic principles or attributes, well developed in the litera-ture[14]. In this article Yin is defined as what is becoming Yang and Yang as what is becoming Yin. Yin and Yang are described respectively with a broken and an unbroken line.

Yin = ▬▬ ▬▬   Yang = ▬▬▬▬

Doubling these two symbols yields the following 4 figures, called bigrams.

▬ ▬	▬ ▬	▬▬▬	▬▬▬
▬ ▬	▬▬▬	▬ ▬	▬▬▬
Old Yin / Winter/ North	Young Yang/ Spring / East	Old Yang / Summer / South	Young Yin /Autumn / West

The deployment of the bigrams in a 4-regions space[15] builds 64 figures composed from Yin and Yang lines, called hexagrams.

Definition 2
A hexagram is called a Wen hexagram when no particular properties are considered. There-fore all of the 64 hexagrams are Wen hexagrams. This is in order to differentiate between first- or second-degree nuclear hexagrams or FuXi hexagrams which are characterized by specific properties.

Definition 3
Each Wen hexagram has a FuXi hexagram given by the trigram-based correspondence be-tween the King Wen and FuXi trigrams' arrangements[16]. A Wen hexagram and its correspond-ing FuXi hexagram are called FuXi related and they belong to the same FuXi family.
This article establishes a mapping between these two arrangements considering some specif-ic hexagram's properties like "nuclear" and "envelope".

Definition 4
A nuclear hexagram is extracted from a hexagram by dropping the first and sixth line and doubling the third and fourth line, it consists of line 2 / line 3 / line 4 / line 3 / line 4 / line 5 of the original hexagram. We distinguish between the first-degree nuclear hexagrams (line 2 = line 4; line 3 = line 5) and the second-degree nuclear hexagrams (line 1 = line 3 = line 5; line 2 = line 4 = line 6). They are the nuclear hexagrams of the nuclear hexagrams. These particular line structures are called the nuclear line structure of first or second-degree.
As an example we consider the hexagram 22 "Grace", its first-degree nuclear is the hexagram 40 "Deliverance"

22   40

Figure 2: The hexagram 22 (left) with its nuclear "Deliverance" (right).
Therefore, there are 16 hexagrams exhibiting a line structure corresponding to a first-degree nuclear hexagram (2 for the line 1 x 2 for the lines 2/4 x 2 for the lines 3/5 x 2 for the line 6) and 4 second-degree nuclear hexagrams (2 (line 1/3/5) x 2 (lines 2/4 /6). The nuclear hexa-

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[14] (Ning Lu, 2008)
[15] East, South, West, North. This deployment will be considered in a forthcoming paper.
[16] Figure 6

grams are not directly involved in the interpretation of a casting, they only help to better understand the real nature of the situation or mutation hexagrams. In this paper the nuclear hexagrams are always orange to distinguish them from the Wen or FuXi hexagrams.

Definition 5
In a hexagram, the lines 1 / 4 or 2 / 5 or 3 / 6 are corresponding if they are pairwise different (Yin/Yang). A hexagram can have 0, 2, 4 or 6 corresponding lines.

Definition 6
The reverse of a hexagram is obtained by reversing the order of the lines.

Hexagram	Reversed hexagram
L6	L1
L5	L2
L4	L3
L3	L4
L2	L5
L1	L6
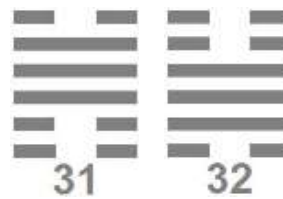
Figure 3: Definition of reverse hexagram. One example hexagrams 31 and 32

Definition 7
The opposite hexagram is obtained by mutating all the lines (Yin becomes Yang, Yang becomes Yin)

Hexagram	Opposite hexagram
Yin / Yang	Yang / Yin
Yin / Yang	Yang / Yin
Yin / Yang	Yang / Yin
Yin / Yang	Yang / Yin
Yin / Yang	Yang / Yin
Yin / Yang	Yang / Yin
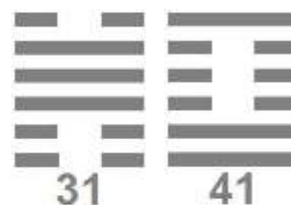
Figure 4: Definition of opposite hexagram. One example hexagrams 31 and 41

Definition 8
The envelope of a hexagram consists of its first and sixth lines. Therefore, there are 4 different envelopes called in this order winter, spring, summer, autumn.

Figure 5: The 4 envelopes

Winter, summer and spring, autumn build two opposite pairs of an envelope.

All the members of a nuclear family have identical inner lines (2/3/4/5), and only differ by their respective envelopes.

Definition 9
A line hexagram results when only one, old-Yin or -Yang, line mutates and the others remain unchanged. Therefore, a casting can have 0,1,2,3,4,5 or 6 accompanying line hexagrams according to the number of changing lines (old-Yin or -Yang lines).

1 Introduction

1.1 The two trigram's arrangements form FuXi and King Wen.

There are traditionally two basic organizations of the 8 trigrams, the FuXi and King Wen arrangements. Each of these illustrates a specific dynamic and the interdependencies between the trigrams.

Figure 6: Pre-heaven (implicate) or FuXi (left circle) and post-heaven (explicate) or King Wen (right circle) arrangements.

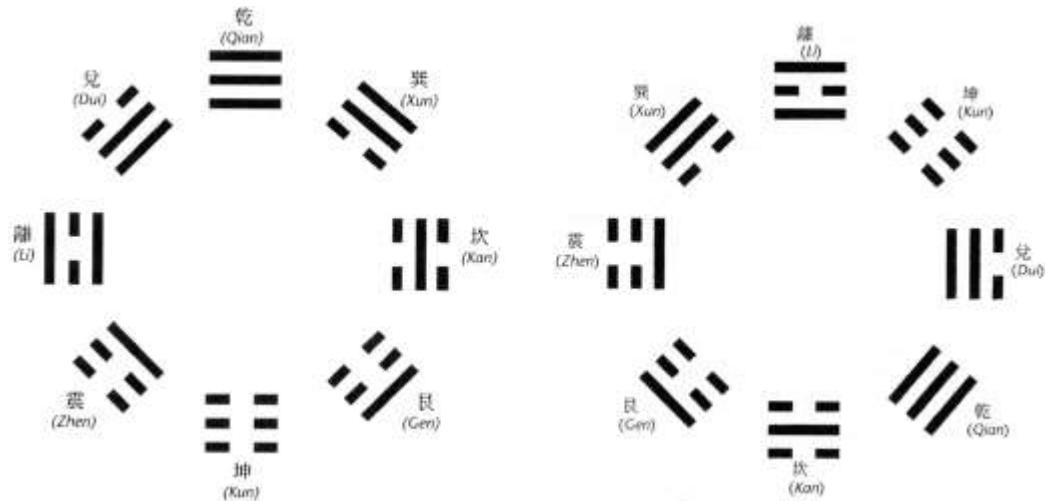The implicate arrangement shows a perfect symmetry of the trigrams, they are diametrically opposed. Considering the explicate arrangement this symmetry only exits for the South/North axis with the Li and Kan trigrams.

The logic of these arrangements has been explained by different authors[17] and we refer to this literature. It is noticeable, that these two arrangements are always described independently from each other, as if they were disconnected. This article aims to demonstrate how closely they are linked together.

1.2 The cyclic nature of the mapping and the 16 FuXi families

The two arrangements correspond to two different permutations of the order of the trigrams. Define P[18] as the matrix defining this permutation. It results that $P^4 = I$ (identity matrix), therefore after 4 transformations the same trigram appears again. Two corresponding trigrams occupy the same place in the respective arrangements. As an example Qian corresponds to Li, Li to Zhen, Zhen to Gen and Gen goes back to Qian.

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[17] (R. Wilhelm, 1961); (Schlumberger, 1987) ; (Schöter, 2011) (More, 1989)
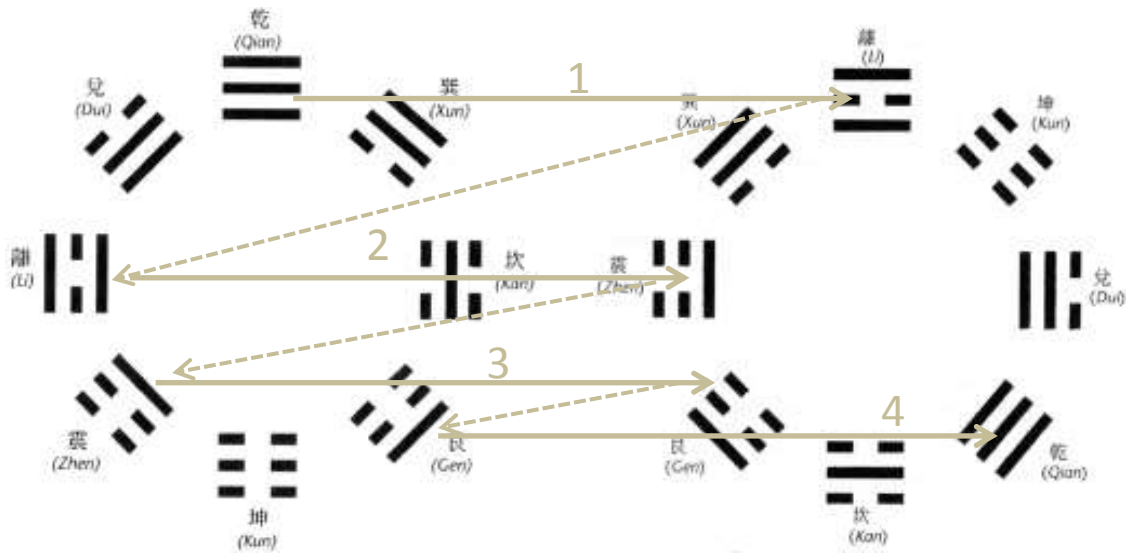[18] See Section 8.1 The permutation matrix between the King Wen and FuXi arrangements

Figure 7: Illustrating the 4-cyclicity of the FuXi and King Wen arrangements with the trigram Qian.

There are 8! (=40'320) distinct sequences to align the 8 trigrams differently. Each of these sequences corresponds one-to-one to a permutation matrix. Therefore it is relatively easy to convince oneself that the 4-cyclicity is not realized for any permutation[19]. Section 9.1 displays the permutation matrix linking the arrangement of King Wen and FuXi. The following illustrations show the 16 different FuXi families cyclically arranged.

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[19] The 4-cyclicity is probably linked with the endless motion of old Yin / young Yang / old Yang / young Yin / old Yin.

1.3   The hexagrams and their 16 FuXi families

All the families, displayed below, have been derived using the correspondences between the two trigrams' arrangement forms FuXi and King Wen and a reordering of the hexagrams as displayed in Figure 7.
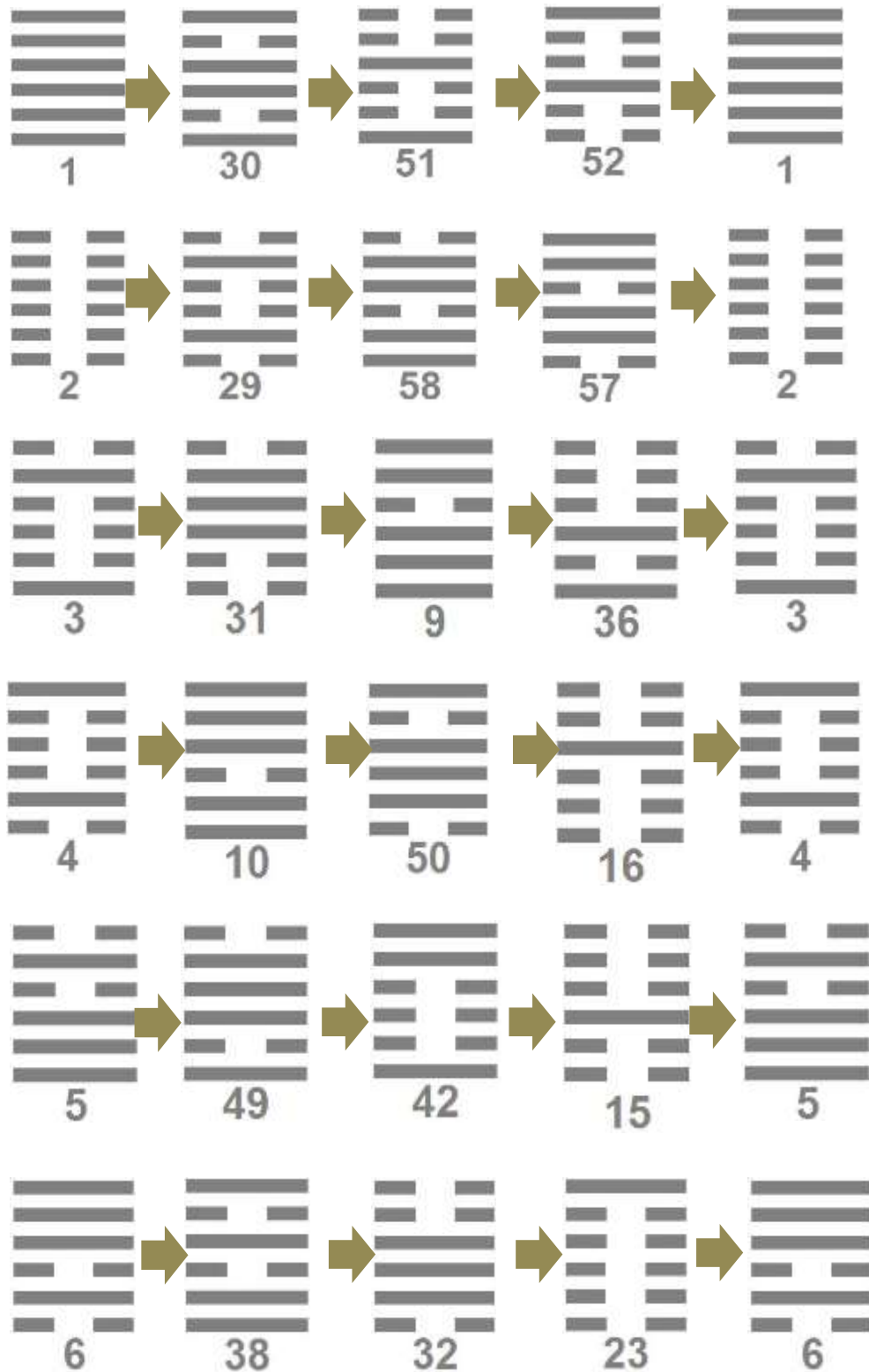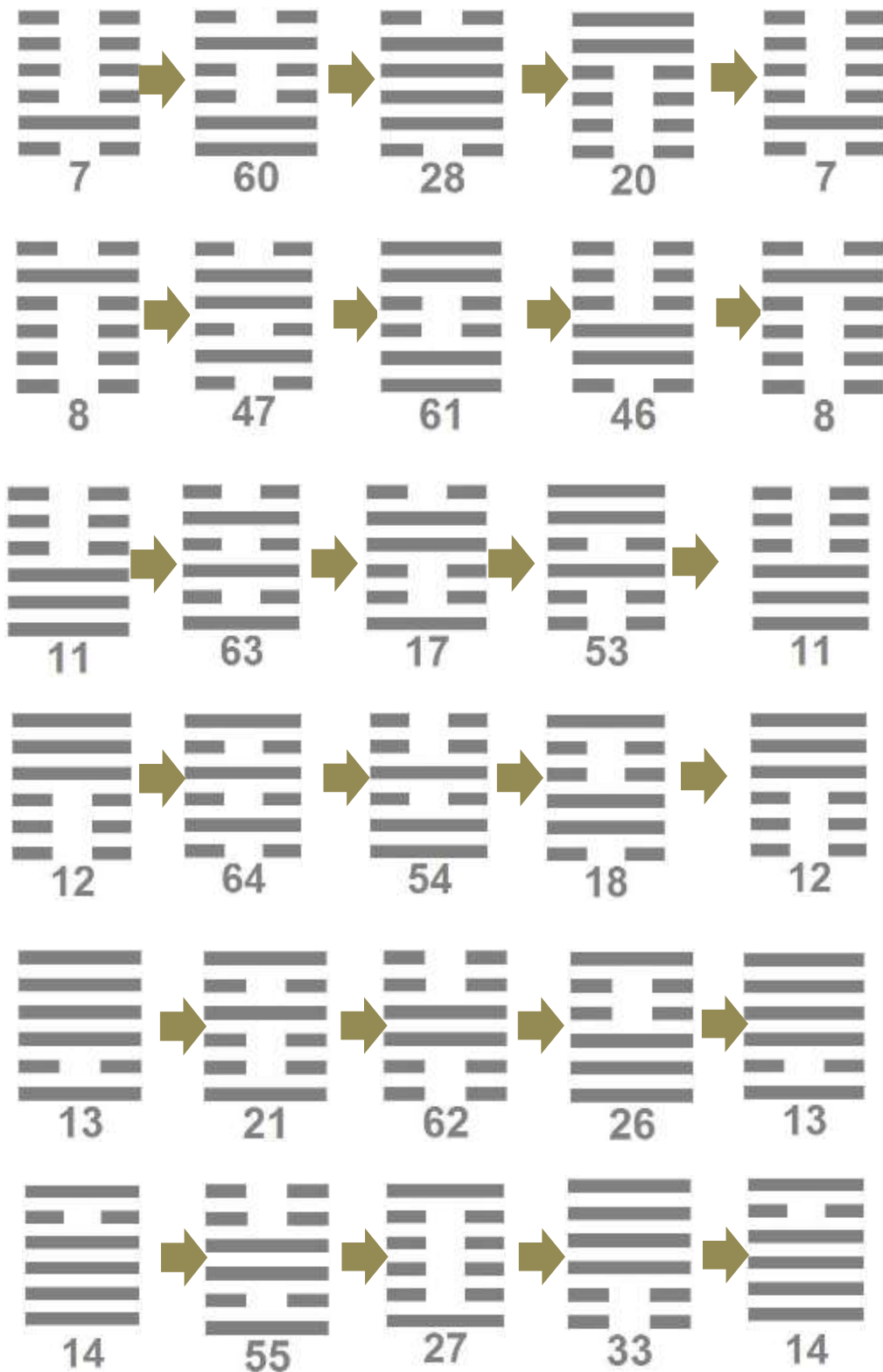
Figure 8: The 16 FuXi families.

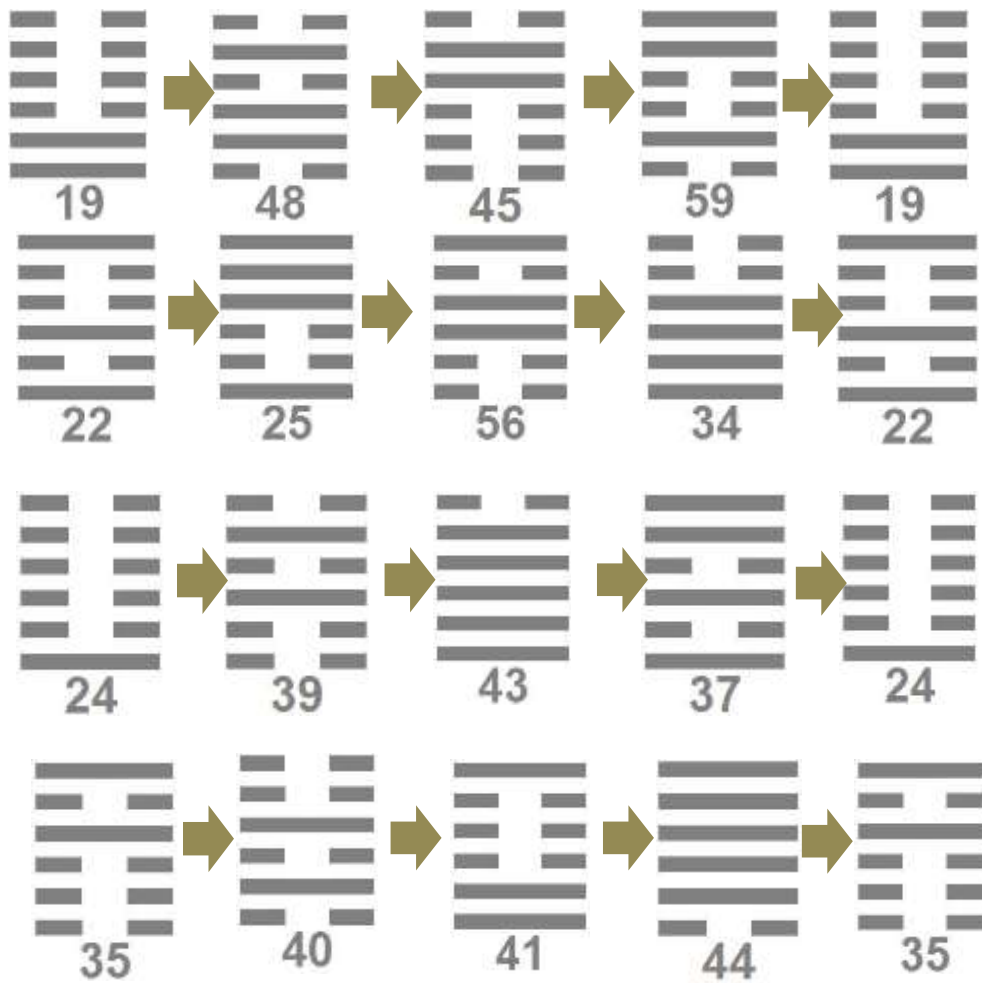## 1.4   The implicate casting and the FuXi hexagrams

As explained in the abstract, the explicate casting[20] generates the situation hexagram with its changing old Yin and old Yang lines, defining the mutation hexagram.

The implicate casting results in transforming the situation and mutation hexagrams according to the King Wen and FuXi trigrams' arrangements. The building trigrams of the explicate hexagrams in the King Wen arrangement correspond to the implicate trigrams having the same position in the FuXi arrangement, thus delivering the FuXi hexagrams for the situation and mutation hexagrams.

It is mandatory to ascertain a precise analysis of the considered situation to clearly understand the specific meaning of the cast hexagram and its relations with the others. The whole set of 64 hexagrams acts as a network, and every hexagram plays a well differentiated role in this network. These roles are implicitly contained in the structure of the hexagram and in the different connections linking the hexagrams. Because each hexagram reflects a precise situation, a part of the interpretation consists in delimiting the semantic of each hexagram from the others.

The Yijing organizes all its hexagrams as 4-member families.

1. The 16 first degree nuclear families grouping 4 hexagrams which share common attributes.
2. The 16 FuXi families, where the hexagrams are either preceding or succeeding[21] in a circular chain.

The nuclear structure is not always obvious but in fact less complicated to understand than the FuXi structure. In nuclear families the relations are somehow static, in a FuXi family there is a circular information flow which is continuously traversing the 4 members, feeding them with specifically needed inputs which trigger some actions or events. What unifies the 4 hexagrams in a nuclear family is the nuclear hexagram considered as a source of attributes for the other family's members.

The FuXi families' signification lies more in what differentiates two hexagrams. The pre-heaven and post-heaven hexagrams are pairwise related through specific relations defined by the opposition of the post-heaven hexagrams. This opposition is translated and expressed through the choice of 2 members of particular nuclear families having opposite envelopes; the latter hexagrams are then the related FuXi hexagrams. Thus indicating and delivering attributes, the post-heaven hexagrams have to integrate or realize some prerequisite conditions requested or recommended by its predecessor in the FuXi family. Two FuXi related hexagrams are bonded by a "predecessor-successor" like relation, the pre-heaven or FuXi hexagram somehow supplies the necessary qualifications which the post-heaven hexagram must own. In a pair of opposite hexagrams, one expresses complementary dimensions which are absent in the other.

A further article will propose an attempt to analyze the different FuXi families as distinct semantic objects and their importance in the interpretation of castings.

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[20] using coins or the Achilles or Yarrow stalks
[21] „The meaning of a word precedes the sound conveying it" (Schlumberger, 1987) Page 52.

2   Definition of the envelope families

2.1   The first- and second-degree nuclear hexagrams and the envelope families

According to the definition of a nuclear hexagram[22], there are 16 hexagrams corresponding to the structure of a first degree nuclear hexagram and each first degree nuclear hexagram builds a first degree nuclear family assembling 4 hexagrams which differ only by their specific envelopes.

Each of the 4 second degree nuclear hexagrams generates its own 4-hexagrams family and the family's members are all first-degree nuclear hexagrams and differ only through the envelope configuration inside their own family.

Figure 9: The 4 second-degree nuclear families.

The four second degree nuclear hexagrams condense some basic properties common to all the respective family members, which as first-degree nuclear hexagrams also condense some basic properties of their family members. All the nuclear hexagrams, first- and second degree may be considered as semantic consolidators.
Inside a second-degree nuclear family and considering the inner lines 2/3/4/5 the 4 hexagrams are identical, only the respective envelopes differentiate between members.

The 4 envelopes act as differentiators in a family and are assemblers across the 4 considered second-degree nuclear families. Gathering together all the hexagrams having the same envelope generates 4 new families called envelope families:

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[22] Definition 4

The South, North, West and East envelope families.

South	North	East	West

Figure 10: The 4 envelope families

These families own the following properties:
1. Members of the same family differ from each other through the inner lines (2/3/4/5).

2. Opposite envelopes generate opposite families. The members of the South-family have their opposite in the North-family and the same relations hold for the members of the East- and West-families.

3. Mutating the envelopes' lines (one or two) maps one family into another family.

4. Each member of an envelope family is a first degree nuclear hexagram and therefore can be considered as a wildcard for 4 hexagrams differing from each other through their specific envelope but exhibiting the same lines configuration (2/3/4/5).

3   Definition of the FuXi nuclear pairs and FuXi hexagrams

3.1   The FuXi nuclear pairs

Each member of the 16 envelope families is representative for 4 hexagrams as a first-degree nuclear hexagram.

Pairing every member of the South-envelope family with one member belonging to the North-envelope family delivers 16 pairs of first-degree nuclear hexagrams[23], the same occurs when pairing every member of the East-envelope family with members of the West-envelope family which delivers 16 pairs of first-degree nuclear hexagrams. These pairs of first-degree nuclear hexagrams are called the **FuXi nuclear pairs**. To facilitate the representation of these combinations, the hexagrams are replaced by their corresponding numbers, therefore the newly associated pairs look like this:

South	North
1	2
1	28
1	39
1	40
27	2
27	28
27	39
27	40
37	2
37	28
37	39
37	40
38	2
38	28
38	39
38	40

West	East
23	24
23	43
23	54
23	63
44	24
44	43
44	54
44	63
53	24
53	43
53	54
53	63
64	24
64	43
64	54
64	63

Table 1: Pairings of the envelope-families

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[23] 4 of them will also satisfy the conditions to be a second-degree nuclear hexagram

3.2   The FuXi hexagrams

The members of the envelope families are all first degree nuclear hexagrams, representing 4 hexagrams linked together by the same nuclear hexagram[24]. Table 2 shows the logic of pairing the members of the South-envelope family belonging to hexagram 1 ("The Creative") with the members of the 4 North-envelope families. To each member of the first-degree nuclear family generated by hexagram 1 exactly one member of one of the four first-degree nuclear families generated by the hexagrams 2/28/39/40[25] corresponds. The mapping of the South-envelope family corresponding to the hexagram 1 produces 4 pairs of hexagrams. Each of these pairs consists of one hexagram belonging to the first-degree nuclear family of hexagram 1 (either hexagram 1 or 44 or 28 or 43) and the second one is chosen out of the first-degree nuclear families of hexagram 2/28/39/40, thus delivering 4 pairs of hexagrams called **the FuXi hexagrams**. In this paper the FuXi hexagrams will be of gold-brown color like the paired fields in Table 2 (i.e. 1/ 2, 56/28).

South			North	
Hexagram 1 (as a FuXi nuclear hexagram, represented through its 4 family members)	1	27	Hexagram 2 (as a FuXi nuclear hexagram, represented through its 4 family members)	
	44	23		
	28	2		
	43	24		
	1	30	Hexagram 28 (as a FuXi nuclear hexagram, represented through its 4 family members)	
	44	56		
	28	62		
	43	55		
	1	21	Hexagram 39 (as a FuXi nuclear hexagram, represented through its 4 family members)	
	44	35		
	28	16		
	43	51		
	1	22	Hexagram 40 (as a FuXi nuclear hexagram, represented through its 4 family members)	
	44	52		
	28	15		
	43	36		

Table 2: Detailed pairing for the South-envelope family of nuclear hexagram 1 with the sequence of North-envelope families, represented by 4 nuclear hexagrams 2,28,39,40.

The same algorithm has to be repeated for the first-degree nuclear hexagrams 27/37/38, members of the South-envelope family. The left-side part of table 2 changes accordingly to the first-degree nuclear families defined by the hexagrams 27/37/38 and the right-side part remains the same.

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[24] The first-degree nuclear families are displayed in 9.7
[25] The hexagrams in a family are ordered according to their envelopes Summer / West / Winter / East

At the end, the mapping of all the South-envelope families onto the North-envelope families generates 16 pairs of FuXi hexagrams.

Applying the same algorithm to the West- and East-envelope families, 16 further pairs of FuXi hexagrams are generated. Therefore, the whole mapping creates 32 pairs of FuXi hexagrams. Each of these 32 pairs of hexagrams has to be somehow related with a pair of Wen hexagrams in order to fit the correspondence involved by the King Wen and FuXi arrangements.

3.3   Conditions for the FuXi families

Obviously there are many possible pairings[26]. Supplementary conditions, based on some inherent hexagrams' properties are necessary to set clear requirements for this mapping to ensure a meaningful pairing scheme corresponding to the creation of the FuXi hexagrams given by the arrangements of King Wen and FuXi. Indeed, the three following further specifications are necessary and sufficient to consistently describe the mapping of the FuXi to the King Wen arrangements.

Principle 1.   The FuXi Sudoku

To each pair of FuXi hexagrams corresponds a pair of opposite Wen[27] hexagrams. Table 3 illustrates this requirement and deserves a detailed explanation. In the left two cells there are two opposite Wen hexagrams XY, the two cells in the middle contain a pair of FuXi hexagrams, each of them is a member of a nuclear family defined in the cells to the right, which contain a pair of FuXi nuclear hexagrams. The two colors indicate the relations existing between the 6 hexagrams. Table 3 shows how the opposition of two Wen hexagrams (explicate casting) is translated in the implicate casting. The property of opposition between the hexagrams in the explicate casting is "smoothed or nuanced" in the implicate casting[28]. A FuXi nuclear pair is a pair of first-degree nuclear hexagrams with opposite envelopes and each of the corresponding FuXi hexagrams belong to one of these FuXi nuclear families[29].

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[26] The evaluation of the number of possible pairings is given in 9.3

[27]   Definition 1

Yin and Yang are considered as two basic principles or attributes, well developed in the literature. In this article Yin is defined as what is becoming Yang and Yang as what is becoming Yin. Yin and Yang are described respectively with a broken and an unbroken line.

Yin = ▬▬   ▬▬   Yang =

Doubling these two symbols yields the following 4 figures, called bigrams.

The deployment of the bigrams in a 4-regions space builds 64 figures composed from Yin and Yang lines, called hexagrams.

Definition 2

[28] Compare sections 7.1 and 7.2

[29] There are still 32 pairs of Wen hexagrams to pair with 32 FuXi pairs (496 possible pairings)

Pair of opposite Wen hexagrams		FuXi pair of hexagrams		FuXi nuclear pair of hexagrams	
XY	Opposite of. XY	FuXi to XY	FuXi to Opp. XY	Nuclear of FuXi to XY	Nuclear of Opp. FuXi to XY

Table 3: Opposite Wen hexagrams and their corresponding pair of FuXi hexagrams and FuXi nuclear hexagrams.

Principle 2.   Inverse free
A FuXi family never contains two opposite hexagrams.

Principle 3.   FuXi invariance
Mutating all the corresponding lines of all the hexagrams of a FuXi family is still a FuXi family. There are families[30] mapped onto other families or onto themselves, especially FuXi families whose hexagrams do not have corresponding lines are considered as self-reflecting.

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[30] Compare section 6.2 Verification of the Principle of FuXi invariance

Principle 4.   4-Cyclicity
The FuXi families have a 4-cyclic pattern.

        H1   ➡   H2   ➡   H3   ➡   H4   ➡   H1
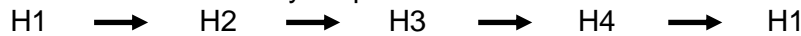
Hexagram H1 is the FuXi of hexagram H2 this is the FuXi of hexagram H3 which is the FuXi of hexagram H4 which closes the loop as the FuXi of hexagram H1.

In section 9.1 the different 8 x 8 permutation matrices Ps are analyzed. Especially the condition P4 = Identity, which is necessary to insure a 4-cyclic pattern.

4   Special groups of hexagrams

4.1   Two FuXi families without corresponding lines

According to Definition 5 the 8 hexagrams, obtained by doubling the 8 trigrams do not have corresponding lines. Therefore (Principle 3) they compose two self-reflecting FuXi families.
A FuXi family does not have two opposite hexagrams (Principle 2), thus there are pairs of hexagrams which do not belong to the same families, explicitly the pairs 1/2, 29/30, 51/57, 52/58.
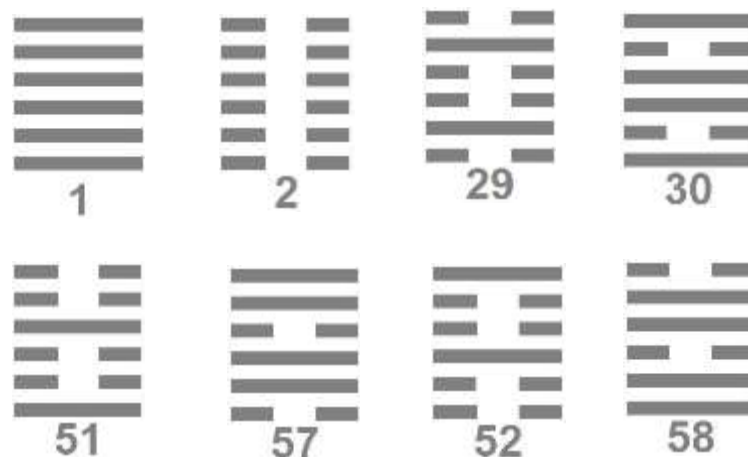
Figure 11: The 8 hexagrams without corresponding lines

While it is clear that these eight hexagrams form two FuXi families, there are still many different ways to select the members of each family and to arrange the family in the proper order. This question is being treated in chapter 5.
It is noticeable that the excluding hexagrams' pairs are formed by either exclusively Yin trigrams (for the hexagrams 2/30/57/58) or exclusively Yang trigrams (for the hexagrams 1/29/51/52).
Although this fact may be interpreted as the Yin-Yang polarity applied to configure the FuXi families, it is shown in chapter 5 that in each of these two families, there are three hexagrams made of Yin or Yang trigrams and one based respectively on Yang or Yin trigrams. One family is realized by doubling the three Yang trigrams Qian, Gen, Zhen, and the Yin trigram Li (hexagram 30) and the other by doubling the Yin trigrams Kun, Xun, Dui and the Yang trigram Kan (hexagram 29). In a predominantly Yin or Yang family, there is a representative for the Yang or Yin attributes.

This structure replicates very well the Yin Yang logo, illustrating the wholeness of the Yin-Yang pair bearing each their own nourishing opposite in an endless loop.

Figure 12: The seeding or analogically fractal property of Yin Yang.

This dynamic Yin-Yang creating process may be recognized in the 4-cyclic structure of the FuXi families, considering the FuXi hexagram like a "seed" for the corresponding Wen hexagram. Clearly this is a speculative statement, but it seems plausible and it reinforces the coherence of the Yijing's internal logic. It also contributes to better understand the role and power of the implicate hexagrams to interpret a casting.

4.2   Two further FuXi families with special properties

In section 9.5 hexagrams, whose reverse and opposite hexagrams are identical, are displayed. Applying the Principles 2 and 3 to this group of hexagrams, it is clear that opposite hexagrams do not belong to the same family and the mutation of the corresponding lines switch from one family to the other. Using these properties, it is possible to pair these 8 hexagrams so that each member of a pair belongs to a different FuXi family.
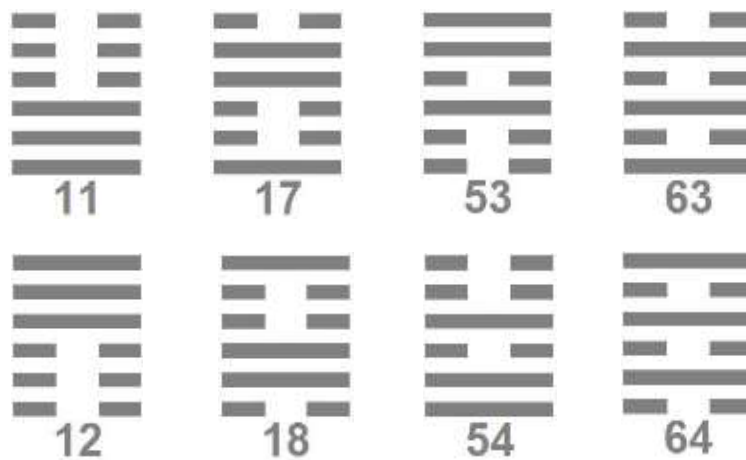
Figure 13: Pair-wise excluding hexagrams for the FuXi families.

The hexagrams 11/17/53/63 build one FuXi family and the hexagrams 12/18/54/64 the second one. Each hexagram in the first group has its opposite in the second group and the mutation of all the corresponding lines of the hexagrams in the first group just map into the second group. In chapter 5, these two groups of hexagrams are arranged in the correct order. This is defining the mapping of the eight trigrams to themselves and therefore recovering the King Wen and FuXi trigrams' arrangements.

5   Assigning each hexagram to its proper FuXi family

In the sections 4.1 and 4.22 hexagrams were identified, which belong to the same families like the hexagrams1,2,29,30,51,52,57,58 for one group and the hexagrams 11,12, 17,18,53,54,63,64 for a second group. They build four distinct FuXi families.
Sections 4.1 and 4.2 organize these 16 hexagrams in 8 pairs of opposite hexagrams, thus belonging to four different families. Next, all these hexagrams will be consistently organized in families using the Principles 1 to 4 defined in 3.3.

5.1   The FuXi nuclear pairs of hexagrams 1/2 the Creative, The Receptive, hexagrams 53/54, Gradual Progress, The Marrying Maiden, hexagrams 30/29, Fire, The Abysmal hexagrams 64/63 Before, After Completion and their generated FuXi hexagrams.

Following Principle 1

Table 3 may be organized as follows with the hexagram 1 and 2 as a pair of FuXi nuclear hexagrams (Table 1).
In 4.1 it is explained that the hexagrams 1,2,29,30,51,52 and 57, 58 build two FuXi families.
Hexagrams 1 and 2 are FuXi nuclear hexagrams and are placed accordingly into the appropriate cells to the right.
The two cells in the center receive respectively one member of the first-degree nuclear family structured by the hexagrams 1 and 2. Hexagrams 1 and 2 belong to the considered nuclear families and also have to belong to the generated FuXi families, thus they are the only suitable candidates to occupy these cells.
The left cells are filled with two opposite hexagrams. It has been established in 4.1 that the hexagrams 29 and 30 belong to different families, formed with the eight hexagrams without corresponding lines, they also are reciprocally opposite and therefore satisfy Principle1. [31]

Pair of opposite Wen hexagrams 30/29		FuXi pair of hexagrams 1/2		FuXi nuclear pair of hexagrams 1/2	
30	29	1	2	1	2

Table 4: FuXi Assignment for the hexagrams 30 and 29

The restrictions explained in 4.2 set further conditions for the mapping. Within the eight hexagrams considered there, which form two FuXi families, there are two FuXi nuclear pairs[32] formed with respectively the hexagrams 53/54 and the hexagrams 63/64. The hexagrams 12 and 11 belong to the first-degree families linked respectively to the hexagrams 53 and 54. The hexagrams 1, 2,11,12, own the same trigrams, Qian and Kun. The hexagrams 29,30,63,64 also own the same trigrams, Kan and Li. In both cases the trigrams are interchanged.
The FuXi mapping is trigram's based, therefore in order to ensure consistency in the mapping, the following disposition is requested.

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[31] The choice of this order of hexagrams 30 and 29 is discussed later in section 0.
[32] Section 3.1 The FuXi nuclear pairs

Contents

Pair of opposite Wen hexagrams 64 / 63		FuXi pair of hexagrams 12 / 11		FuXi nuclear pair of hexagrams 53 / 54	
64	63	12	11	53	54

Table 5: FuXi assignment for the hexagrams 64 and 63.

The trigrams Qian and Kun in FuXi arrangements correspond to the trigrams Li and Kan in Wen arrangements, as already mentioned in section 4.11.

5.2 The FuXi nuclear pairs of hexagrams 53/63, Gradual Progress, After Completion, hexagrams 54/64, The Marrying Maiden, Before Completion, hexagrams 63/64 and their generated FuXi hexagrams.

According to Table 1 the pairs of first-degree nuclear hexagrams 53-63, 54-64 and 64-63 also build FuXi nuclear pairs. Therefore using Principle 1, these three pairs of first-degree nuclear hexagrams can be organized as shown in the following table.

Pairs of opposite Wen hexagrams X/X-X/X-X/X[33]		FuXi pairs of hexagrams X/X-X/X-X/X		FuXi nuclear pairs of hexagrams 53/63-54/64-63/64	
				53	63
				54	64
				63	64

Table 6: The FuXi nuclear pairs 53-54, 54-64 and 63-64

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[33] X is a wildcard for a not yet defined hexagram. "/" indicate left/right column, "-" indicates a row's change

In section Two further FuXi families with special properties4.2 it is explained, that the hexa-grams 11,12,17,18,53,54,63, 64 form two FuXi families. The links between the hexagrams 11,12,63,64 has been shown in 5.1. The hexagrams 63 and 64 have as FuXi hexagrams re-spectively the hexagrams 11 and 12 (

Table 5).
The corresponding FuXi hexagrams for the remaining hexagrams 11,12,17,18 and 53,54 still have to be determined.

The hexagrams 17 and18 are members of the first-degree nuclear hexagrams 53 and 54, therefore their respective places in Table 6 are uniquely defined as shown in the next table.
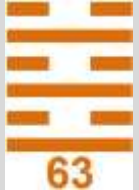
Pairs of opposite Wen hexagrams X/X-X/X-X/X		FuXi pairs of hexagrams 17/X-18/X-X/X		FuXi nuclear pairs of hexagrams 53/63-54/64-63/64	
		17		53	63
		18		54	64
				63	64

Table 7: The FuXi nuclear pairs 53-54, 54-64 and 63-64 and the hexagrams 17,18

The hexagrams 17 and18 are not only reverse but also opposite to each other. Following Principle 1, they have to be positioned in two cells in the first two columns of Table 7. Principle 2 forbids having two opposite hexagrams in a FuXi family; therefore the only suitable locations are the two left cells in the third row:
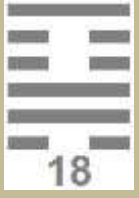
Pairs of opposite Wen hexagrams X/X-X/X-18/17		FuXi pairs of hexagrams 17/X -18/X-X/X		FuXi nuclear pairs of hexagrams 53/63-54/64-63/64	
		17		53	63
		18		54	64
18	17			63	64

Table 8: The FuXi nuclear pairs 53-54, 54-64 and 63-64 and the hexagrams 17,18.

In section 4.2 it is explained that hexagram 12 and 18 belong to the same FuXi family, thus hexagram 12 is placed as shown in

Table 9. To place hexagram 12, immediately positions hexagram 11 as the opposite of hexagram 12.

Pairs of opposite Wen hexagrams X/X-12/11 – 18/17		FuXi pairs of hexagrams 17/X–18/53-X/X		FuXi nuclear pairs of hexagrams 53/63-54/64-63/64	
		17		53	63
12	11	18	53	54	64
18	17			63	64

Table 9: The FuXi nuclear pairs 53-54, 54-64 and 63-64 and the hexagrams 11,12

The FuXi hexagram of hexagram 12 is hexagram 18. The FuXi hexagram of hexagram 11 is necessarily hexagram 53, because hexagram 53 and 11 are, as explained in section 4.2, in the same FuXi family and hexagram 64 is the first-degree nuclear of hexagram 53.

Still following the reasoning deployed in section 4.2 for the third row, the pairs of hexagrams 17/63 and 18/54 are respectively in the same FuXi families. Hexagram 63 belongs to the first-degree nuclear family of hexagram 64 and hexagram 54 to that of hexagram 63.

Pairs of opposite Wen hexagrams X/X-12/11-18/17		FuXi pairs of hexagrams 17/X – 18/53- 54/63		FuXi nuclear pairs of hexagrams 53/63-54/64-63/64	
		17		53	63
12	11	18	53	54	64
18	17	54	63	63	64

Table 10: The FuXi nuclear pairs 53-54, 54-64 and 63-64 and the hexagrams 63,54

The pair of opposite hexagrams 53-54 occupies the two left cells in the upper corner of

Table 10, and the order is given by the fact that hexagram 17 and hexagram 53 are in the same family, this implies the following arrangement.
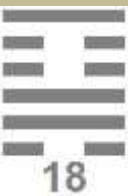
Pairs of opposite Wen hexagrams 53/54-12/11-18/17		FuXi pairs of hexagrams 17/X –18/53-54/63		FuXi nuclear pairs of hexagrams 53/63-54/64-63/64	
53	54	17		53	63
12	11	18	53	54	64
18	17	54	63	63	64

Table 11: The FuXi nuclear pairs 53-54, 54-64 and 63-64 and the hexagrams 53, 54

The hexagrams 54 and 64 lie in the same FuXi family, therefore the FuXi hexagram of 54 is hexagram 64 and the final arrangement is given by the next table.

Pairs of opposite Wen hexagrams 53/54-12/11-18/17		FuXi pairs of hexagrams 17/64–18/53-54/63		FuXi nuclear pairs of hexagrams 53/63-54/64-63/64	
53	54	17	64	53	63
12	11	18	53	54	64
18	17	54	63	63	64

Table 12: The FuXi nuclear pairs 53-54, 54-64 and 63-64 and the hexagrams 54,64

6 Recovering the King Wen and FuXi trigrams' arrangements

6.1 Consolidation of the tables and validation of the results

Consolidating the tables 5 and 12 assign to each of the hexagrams 64,63,53,54, 12,11,18,17 their corresponding FuXi hexagram.

Pairs of opposite Wen hexagrams 64/63-53/54-12/11-18/17		FuXi pairs of hexagrams 12/11-17/64–18/53-54/63		FuXi nuclear pairs of hexagrams 53/54-53/63-54/64-63/64	
64	63	12	11	53	54
53	54	17	64	53	63
12	11	18	53	54	64
18	17	54	63	63	64

Table 13: Consolidated table.

Comparing the hexagrams in the first two left columns with the ones in the third and fourth columns establishes the correspondences between the King Wen and FuXi trigrams' arrangements.

Order Wen (explicate)			Order Fuxi (implicate)		
Qian		1	**Gen**		5
Kun		2	**Xun**		6
Zhen		3	**Li**		7
Kan		4	**Kun**		2
Gen		5	**Zhen**		3
Xun		6	**Dui**		8
Li		7	**Qian**		1
Dui		8	**Kan**		4

Table 14: Translating the correspondences between the hexagrams of Table 13 into trigrams' correspondences.

Organizing

Table 14 in two circles produces again the Figure 6
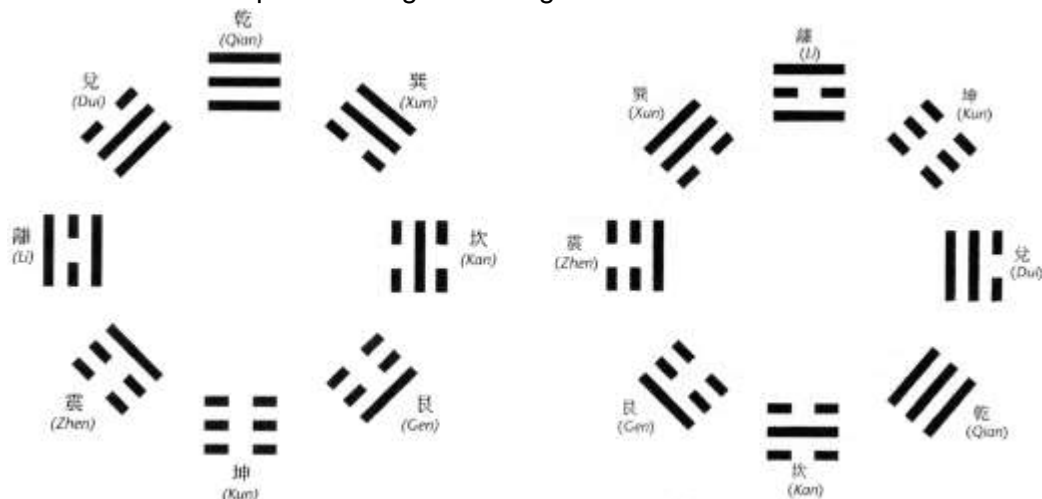
Figure 14: Pre-heaven (implicate) or FuXi (left circle) and post-heaven (explicate) or King Wen (right circle) arrangements.

6.2 Verification of the Principle of FuXi invariance

Figure 8 shows all the 16 FuXi families derived using the original FuXi and King Wen trigrams' arrangements. These families are obviously identical with them, defined by using the four Principles. Table 15 below arranges the FuXi families by bracketed pairs, linked through the mutation of their corresponding lines and four standing-alone FuXi families reflecting themselves by mutating the corresponding lines. Therefore the validity of the FuXi invariance principle is verified.

37
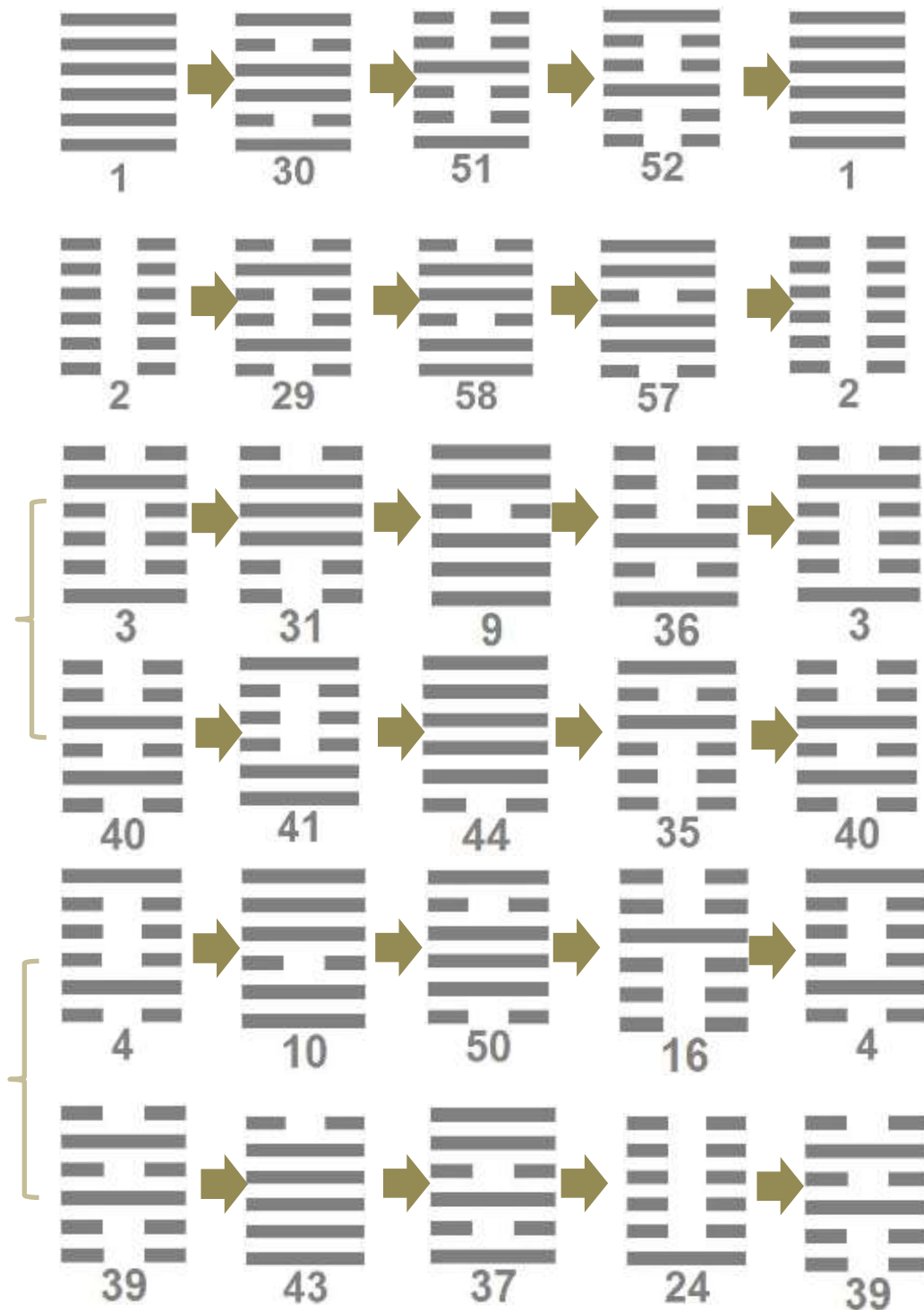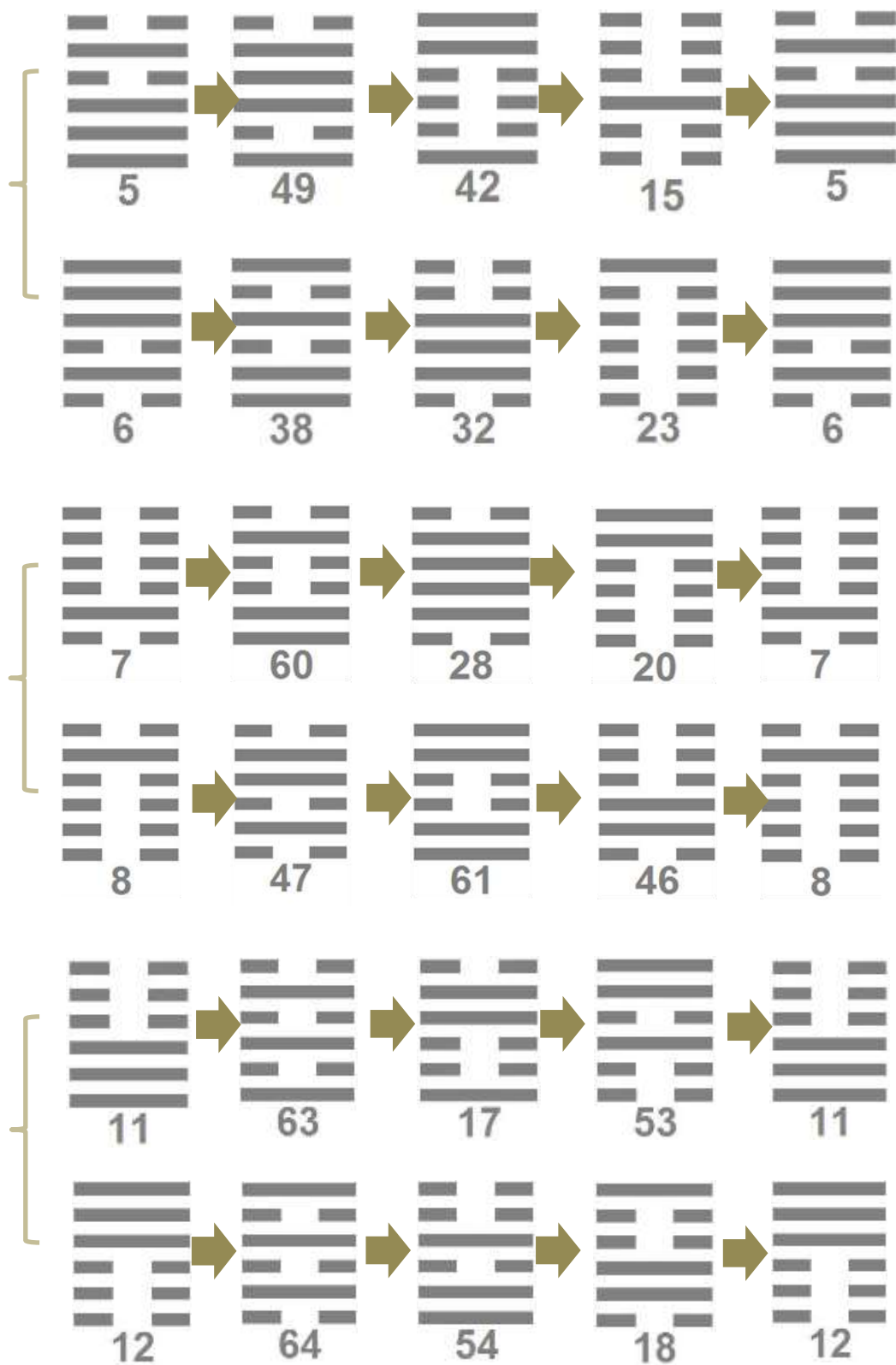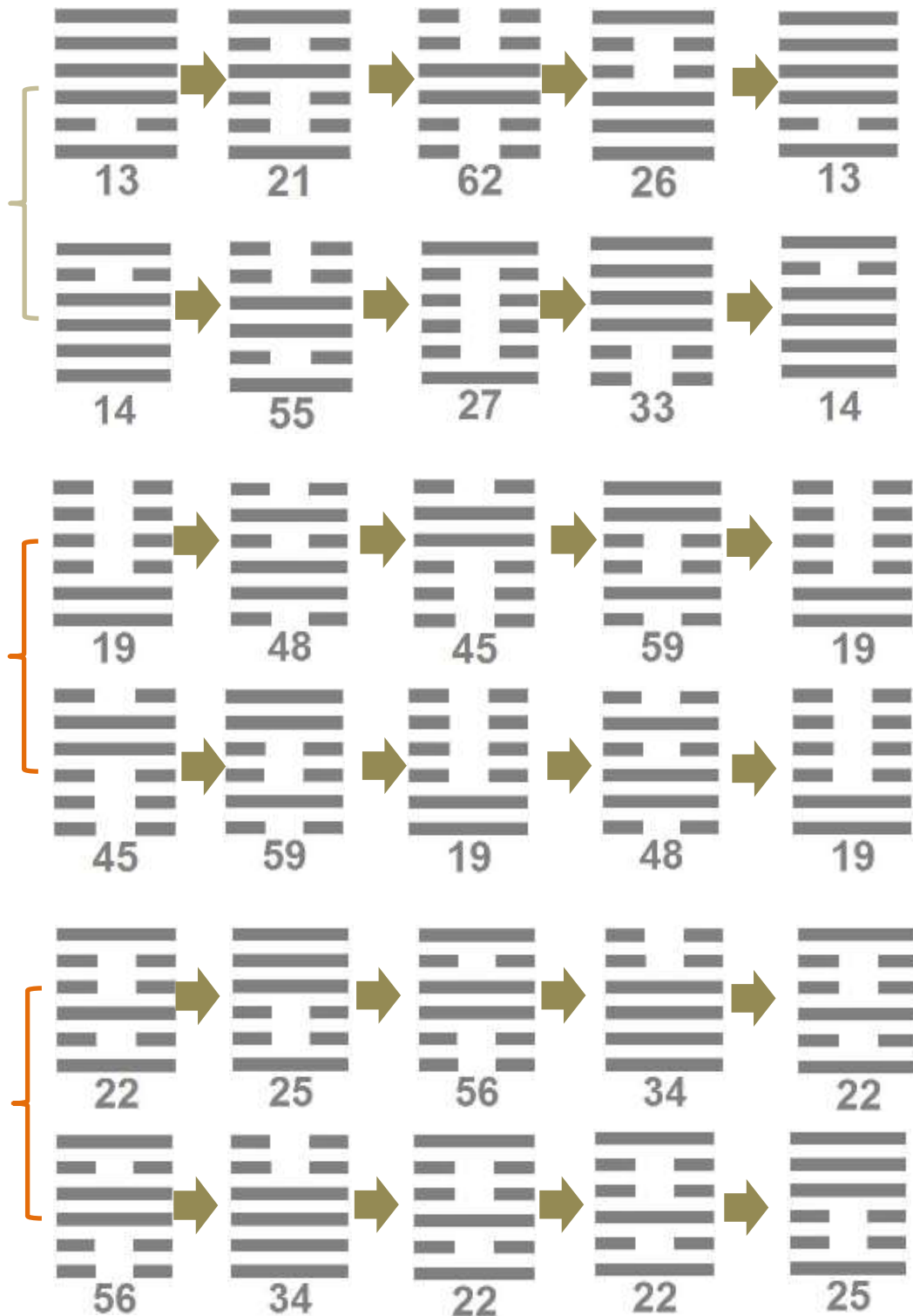
Table 15: The twelve gold-brown bracketed FuXi families arranged by pairs, linked through the mutation of their corresponding lines, and two orange bracketed FuXi families reflecting themselves with a shift and the two first families (hexagrams 1, 30, 51, 52,1 and hexagrams.2, 29, 58,57)without move.

7   Conclusion and further investigations

7.1   The four-valued logic underpinning the Yijing

Among the 64 hexagrams there are some couples with a strong polarity.
For example hexagrams 1,2,11,12,31,32,41,42,63,64.

Figure 15: A special sequence of pairs of hexagrams

These sequences may be considered as a line of growing complexity in the hexagrams' specific logic. Starting with hexagrams 1 and 2, equipped with a unifo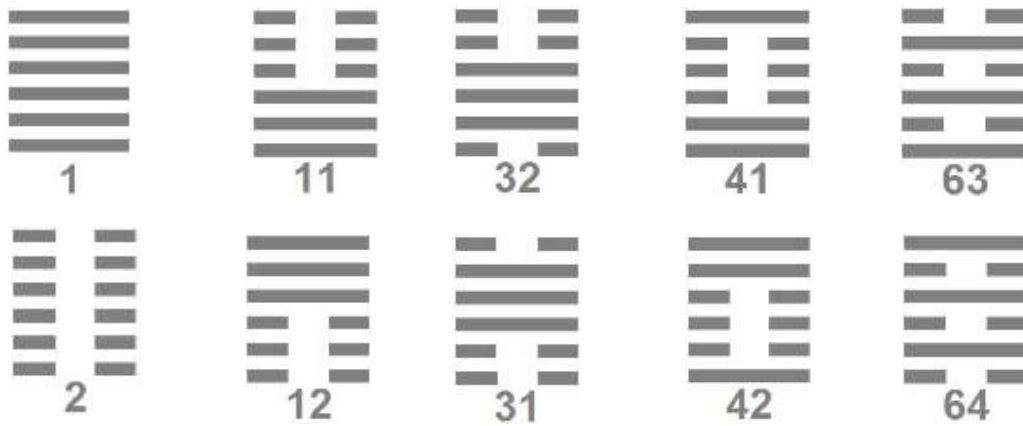rm, erratic Yang and Yin logic, this sequence goes through different states to end with hexagrams 63 and 64 both displaying a sophisticated intricacy
The process is launched with the re-arrangement of the trigrams Qian and Kun, creating six corresponding lines, which generate the two-cyclic dynamic of hexagrams 11/12. This dynamic culminates in the delicate interplay of the hexagrams 63 and 64, the last closes the Book of Change igniting a new organization.

Figure 16: Cyclic dynamic of hexagrams 11/12, 31/32, 41/42, 63/64

This entanglement of the hexagrams is deployed by means of mutating the corresponding lines of the pair of hexagrams 11/12. Mutating the lines 1 and 4 produces hexagrams 32/42, mutating the lines 3 and 6 produces hexagrams 41/31 and mutating the inner lines 2 and 5 results in the last two hexagrams 63 and 64. While the hexagrams 31,32,41,42 are related to each other being either reverse or opposite and the hexagrams 11 and 12 are reverse and opposite[34], the hexagrams 63/64 are the only two consolidating all these attributes with the fact that they are the reciprocal first-degree nuclear hexagrams and identical to their second-degree nuclear hexagrams, the last property being also shared by the hexagrams 1 and 2. Thus the four hexagrams 1, 2, 63, 64 play an outstanding role in the Yijing's inherent logical structure. This is based on a four-valued logic defined by the very deep characteristics of each

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[34] Like the hexagrams 17/18 and 53/54. All of them resulting from mutating the lines 3 and 4 or 1 and 6
    from hexagrams 11/12.

of these four special hexagrams. These characteristics will be analyzed more in detail in a forthcoming article at a later date

7.2   Principle of opposition or complementarity

The pre-heaven or implicate hexagrams and post-heaven or explicate hexagrams are related primarily through different relations of opposition. The opposition is explicitly given by:
1. The 32 pairs of opposite Wen hexagrams are related to 32 pairs of first-degree FuXi nuclear hexagrams.
2. The two hexagrams of a pair of FuXi nuclear hexagrams differ at least by their opposite envelope (hexagrams 1/28, 2/27..,), or they are line-by-line opposite.

Each pair of FuXi nuclear hexagrams generates a pair of FuXi hexagrams, each of these belonging to one of the two first-degree nuclear families and they correspond to a pair of opposite Wen hexagrams.
This means that the links between the implicate/pre-heaven and explicate/post-heaven are entirely based on the six-line structure of the hexagrams. The use of the trigrams is extremely helpful to facilitate the determination of the corresponding FuXi hexagram for each given Wen hexagram. The two trigrams' arrangement, form King Wen and FuXi, allow to condensate the large Table 20, listed in section 9.6, but they are not very meaningful to help to understand the role of the implicate/pre-heaven hexagrams.

7.3   Principle of inheritance or seeding

Within a given FuXi family, the cyclic chain of the four hexagrams defines a predecessor and a successor for each hexagram, thus creating a kind of "inheritance" of hexagram specific attributes. The impact of these influences deserves more attention and will be analyzed in a forthcoming article.

$$H1 \longrightarrow H2 \longrightarrow H3 \longrightarrow H4 \longrightarrow H1$$
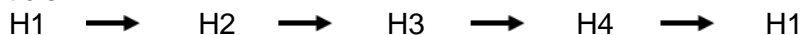
As an example, hexagrams 3 and 50 are considered in more detail. These two hexagrams have a special meaning, because they also divide the Book of Change in two parts. All the hexagrams between these two also have their opposite between them and consequently all hexagrams between the hexagrams 51 and 64 have their opposite in that range[35].

Table 20 gives the allocated pairs of FuXi hexagrams and FuXi nuclear hexagrams.

Pair of opposite Wen hexagrams 3/50		FuXi pair of hexagrams 36/10		FuXi nuclear pair of hexagrams 40/37	
3	50	36	10	40	37

Table 16: Hexagrams 3 and 50 with their FuXi hexagrams

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[35] This fact explains the positions of the hexagrams, "Difficulty in the Beginning" and "The Caldron", in the Book of Change. Hexagram "Difficulty in the Beginning" has to be placed in the right beginning after hexagrams "The Creative" and "The Receptive" therefore it becomes the number 3. Hexagram "The Caldron" as the opposite of hexagram 3 has to be placed at the end of the first part, thus receiving the number 50. This immediately fixes the position of the hexagrams 4 and 49 as the respective reverses of hexagrams 3 and 50.

The opposition of the Wen hexagrams 3 and 50 corresponds to the opposition of the FuXi nuclear hexagrams 40 and 37. Noticeable is the fact that the two FuXi hexagrams 36 and 10 are almost opposite each other. Only the line 1 does not fulfill the right conditions.

The hexagrams 3 and 50 both describe situations of qualitatively different renewals, the hexagrams 36 and 10 explain qualitatively different behavioral attitudes rooted in the opposite nuclear hexagrams 40 and 37, most suitable to achieve the respective goals of the hexagrams 3 and 50.

To investigate all these interdependencies between the hexagrams belonging to the same FuXi family and their assigned FuXi nuclear pairs will be the central theme of forthcoming publications. This example aims to demonstrate the possible benefits derived from this consideration of the generation process for the FuXi hexagrams.

7.4 The FuXi hexagrams as marks for the Book of Changes

It is usual to consider the hexagrams 1 to 30 as the first part of the Book of Change and the remaining hexagrams 31 to 64 as its second part. Each part contains 18 different hexagram figures, some of them can be reversed; some cannot.

The hexagrams 1 and 2 are respectively the FuXi hexagrams of the hexagrams "Fire" and "Abysmal"[36], and they close the first part of the Book of changes.

Further[37] hexagram 3, "The difficulty of the Beginning", is the FuXi of hexagram 31, "Influence", the hexagram initiating the second part of the Book of Changes.

Pair of opposite Wen hexagrams 31/41		FuXi pair of hexagrams 3/40		FuXi nuclear pair of hexagrams 23/63	
31	41	3	40	23	63

Table 17: FuXi mapping for hexagrams 31 and 41

This correspondence between the implicate or pre-heaven order and the partitions of the Book of Changes demonstrates again how smartly, precisely and differentiated the Yijing methodology was conceptualized with the support of the hexagram's structure! How far may the FuXi arrangement help to understand the organization of the Book of Changes?

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[36] Table 20.

[37] Table 20

7.5 FuXi hexagrams and nuclear families

A further prospective field deserving more attention is to consider identifiable relations be-
tween the hexagrams coupled with FuXi hexagrams rooted in the same nuclear family. As an
example see hexagram 64.

Pairs of opposite Wen hexagrams 11/12-17/18-24/44-43/23		FuXi pairs of hexagrams 53/18,63/54,37/41,39/32		FuXi nuclear pairs of hexagrams 64/54,64/63,64/24,64/43	
11	12	53	18	64	54
17	18	63	54	64	63
24	44	37	41	64	24
43	23	39	32	64	43

Table 18: The FuXi nuclear family and its related hexagrams

What kind of characteristics has the hexagrams 11,17,24,43 in common? What happens when
hexagram 63 substitutes hexagram 64? This is a large set of questions, whose answers will
help to enhance the understanding of the rules of change.

7.6 To summarize

A hexagram consists of a structured figure of six-lines and specific semantic contents. Moving
from one hexagram's geometry to another one moves in a "parallel manner" the semantic con-
tents. This is no doubt one of the most interesting characteristic features of the Yijing.
The network of the FuXi families interferes strongly with the nuclear families. The perspective
of looking at the structural roots of the pre-heaven or implicate hexagrams developed in this
paper opens new ways to better understand the links between the hexagrams and to more
appreciate the semantic identity of each of them. This results in an enhanced analysis and
better groundwork to validate the strategy suggested by the casting and therefore to help
promoting the Yijing as a valuable tool for decision making in a managerial environment.

8 Literature

Bohm David Wholeness and the Implicate Order [Buch]. - [s.l.] : Routledge Kegan Paul, 1980.

Faure Pierre und Javary Cyrille Yi Jing [Buch]. - 2002.

Faure Pierre Le Yijing par lui-même [Buch]. - 2006. - Alphée.

Gisin Nicolas L'impensable hasard [Buch]. - [s.l.] : Odile Jacob, 2012.

Javary Cyrille Le Discours de la Tortue [Buch]. - [s.l.] : Albin Michel, 2003.

Javary Cyrille Understanding the I Ching [Buch] / Übers. McElhearn Kirk. - [s.l.] : Shambhala, Boston, 1997.

More Steve The Trigrams of Han [Buch]. - [s.l.] : The Aquarian Press, 1989.

Ning Lu Ph.D Yin-Yang Code [Buch]. - California : Moutain View, 2008. - S. 24.

R. Wilhelm Richard The I Ching [Buch] / Übers. Baynes. - 1961. - S. 122-134.

Schlumberger J-P Yi King Principes, pratique et interprétation. [Ouvrage]. - [s.l.] : Editions Dangles, 1987. - pp. 66-72.

Schöter Andreas The Yi Jing: Metaphysics and Physics [Artikel] // Journal of Chinese Philosophy. - September 2011. - S. 412-426.

9   Appendices

9.1   The permutation matrix between the King Wen and FuXi arrangements

The two fundamental trigrams' arrangements can be deployed as two different permutations of 8 objects numbered from 1 to 8 as follows:
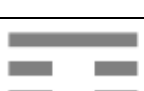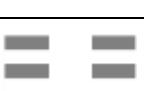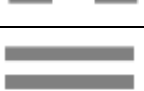
Order Wen (explicate)			Order Fuxi (implicate)		
Qian		1	Gen		5
Kun		2	Xun		6
Zhen		3	Li		7
Kan		4	Kun		2
Gen		5	Zhen		3
Xun		6	Dui		8
Li		7	Qian		1
Dui		8	Kan		4

Table 19: Trigrams' permutations corresponding to the King Wen (explicate) and FuXi (implicate) arrangements

Therefore, the corresponding permutation's matrix is explicitly given by P with

$$
P = \begin{vmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{vmatrix}
$$
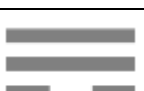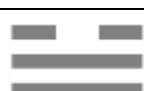
and

$$P^4 \; = \; \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Equation 1 Permutation matrices P and P^4

9.2 Consequences of exchanging the trigrams Li and Kan in the FuXi arrangement

In section 5.1 the trigram Li is assigned to Qian and Kan to Kun. This choice is critical because it ensures the four-cyclicity of the FuXi families. As mentioned in section 4.1 it underlines the intricacy between Yin and Yang, which is the real trigger for the dynamic of change. Exchanging the trigrams Li and Kan in the FuXi arrangement corresponds to a new permutation matrix P* explicitly given by

Order Wen (explicate)			Order Fuxi (implicate)		
Qian		1	Gen		5
Kun		2	Xun		6
Zhen		3	Li		7
Kan		4	Qian		1
Gen		5	Zhen		3
Xun		6	Dui		8
Li		7	Kun		2
Dui		8	Kan		4

$$P^* = \begin{vmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$$
P^{*4} \quad = \quad
\begin{vmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{vmatrix}
$$

So P* does not satisfy Principle 4 and this option has to be discarded. The other trigram assignments are completely defined by the Principles 1 through 3. Therefore, this set of principles is necessary and sufficient to establish the mapping of the King Wen and FuXi trigram arrangements.

Figure 17: The FuXi sequence

This sequence indicates the hexagram 30, "Fire" as the corresponding pre-heaven/implicate hexagram to "The Thunder, The Arousing". This may be interpreted pictorially, considering the lightning as preceding the thunder and the thunder being echoed or absorbed by the mountains.

9.3 Evaluation of the number of possible pairings for the South-envelope family of hexagram 1

South		North	
	1		27
	44	2	23
	28		2
	43		24
	1		30
	44	28	56
	28		62
	43		55
1	1		21
	44	39	35
	28		16
	43		51
	1		22
	44	40	52
	28		15
	43		36

For hexagram 1 there are   16 possible pairing-partners
For hexagram 44 there are 12 possible pairing-partners[38]
For hexagram 28 there are   8 possible pairing-partners
For hexagram 43 there are   4 possible pairing-partners
Therefore, only for the South-envelope family of hexagram 1 we have (16 x 12 x 8 x 4 =) 6144 possible pairings. There is no real interest to know exactly how many different pairings are theoretically feasible. It shows clearly that more conditions have to be explained in order to re-trieve the mapping between the arrangements of King Wen and FuXi.

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[38] Each member of the first-degree nuclear family of hexagram 1 has to be mapped in a different North-envelope family.

9.4   The mutation of the corresponding lines

To mutate all the corresponding lines[39] of any hexagram is like interchanging its lower and upper trigram.

One hexagram may have zero[40], two (1/4 or 2/5 or 3/6), four or six[41] corresponding lines. If the considered hexagram has two corresponding lines, then the lower and upper trigrams differ only on these two lines and mutating them corresponds to changing the trigram's order. The same applies to four or six corresponding lines.

If the hexagram in question has no corresponding lines, then the two trigrams are the same and the exchange is considered as obvious.

Applying this rule to the trigrams' arrangements of Wen and FuXi it is clear why a FuXi family remains a FuXi family even if all the corresponding lines are mutated. Exchanging the trigrams order in the Wen arrangement induces the same move in the FuXi arrangement. Therefore, the mutation of all the corresponding lines of the four hexagrams of a FuXi family necessarily results in another FuXi family.

This property is very helpful to identify the corresponding hexagrams as shown in chapter 5.

9.5   Hexagrams with identical reverse and opposite hexagrams

According to Definition 6 and Definition 7 there are only 8 hexagrams whose opposite is identical with the reverse.

One hexagram with these properties has its sixth, fifth and fourth lines defined by the first, second and third line. There are two possibilities for each of these lines; therefore, there are eight hexagrams with this property and this hexagrams builds together 2 FuXi families linked through the mutation of their corresponding lines.
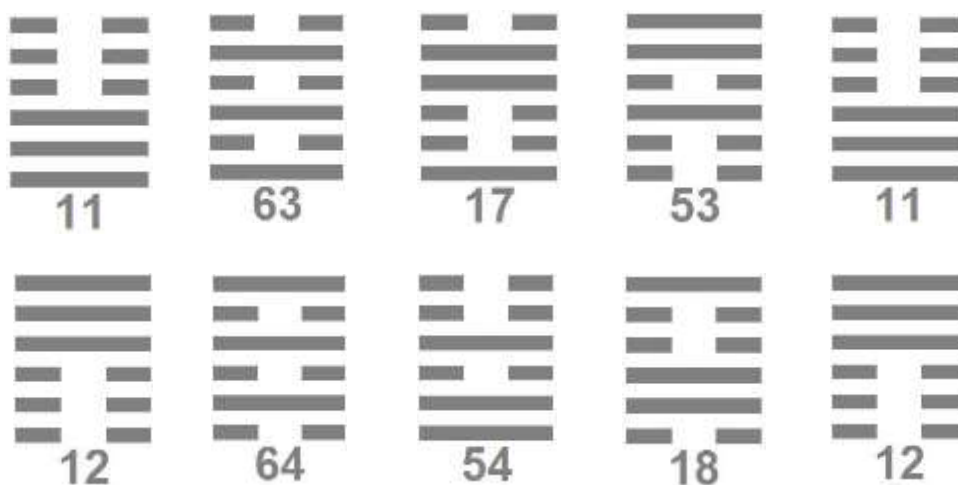
Figure 18: The 8 hexagrams with identical reverse and opposite.

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[39] Definition 5
[40]   This concerns the eight hexagrams obtained by doubling the trigrams: hexagrams 1,2,29,30,51,52,57,58
[41]   Line 1/2/3 completely defines the lines 4/5/6; therefore, there are eight hexagrams with six corresponding lines. The hexagrams 11/12, 31/32 41/42, 63/64. All these hexagrams are derived from 11/12 by mutating the lines 1/4, 3/6 and 2/5 respectively.

9.6 The complete list of the FuXi hexagrams and their related FuXi nuclear pairs

The following table shows all the pairs of opposite Wen hexagrams, the related FuXi hexagrams and FuXi nuclear pairs as formulated in the FuXi Sudoku.

Pair of opposite Wen hexagrams		FuXi pair of Hexagram		FuXi nuclear pair of hexagrams with envelope			
30	29	1	2	1	Summer	2	Winter
20	34	28	56	1	Summer	28	Winter
37	40	43	35	1	Summer	39	Winter
35	5	44	15	1	Summer	40	Winter
29	30	2	1	2	Winter	1	Summer
33	19	27	59	2	Winter	27	Summer
39	38	24	6	2	Winter	37	Summer
6	36	23	9	2	Winter	38	Summer
15	10	42	4	23	Autumn	24	Spring
47	22	8	34	23	Autumn	43	Spring
7	13	20	26	23	Autumn	54	Spring
31	41	3	40	23	Autumn	63	Spring
10	15	4	42	24	Spring	23	Autumn
48	21	19	13	24	Spring	44	Autumn
60	56	7	25	24	Spring	53	Autumn
44	24	41	37	24	Spring	64	Autumn
19	33	59	27	27	Summer	2	Winter
28	27	60	55	27	Summer	28	Winter
58	52	29	51	27	Summer	39	Winter
46	25	61	22	27	Summer	40	Winter
34	20	56	28	28	Winter	1	Summer
27	28	55	60	28	Winter	27	Summer
51	57	30	58	28	Winter	37	Summer
26	45	62	48	28	Winter	38	Summer
38	39	6	24	37	Summer	2	Winter
57	51	58	30	37	Summer	28	Winter
61	62	47	21	37	Summer	39	Winter
50	3	10	36	37	Summer	40	Winter
36	6	9	23	38	Summer	2	Winter
45	26	48	62	38	Summer	28	Winter
49	4	5	16	38	Summer	39	Winter
2	1	57	52	38	Summer	40	Winter
40	37	35	43	39	Winter	1	Summer
52	58	51	29	39	Winter	27	Summer
62	61	21	47	39	Winter	37	Summer
4	49	16	5	39	Winter	38	Summer
5	35	15	44	40	Winter	1	Summer

Pair of opposite Wen hexagrams Hexagram #		FuXi pair of hexagrams Hexagram		FuXi nuclear pair of hexagrams with envelope			
25	46	22	61	40	Winter	27	Summer
3	50	36	10	40	Winter	37	Summer
1	2	52	57	40	Winter	38	Summer
22	47	34	8	43	Spring	23	Autumn
16	9	50	31	43	Spring	44	Autumn
55	59	14	45	43	Spring	53	Autumn
23	43	32	39	43	Spring	64	Autumn
21	48	13	19	44	Autumn	24	Spring
9	16	31	50	44	Autumn	43	Spring
14	8	33	46	44	Autumn	54	Spring
42	32	49	38	44	Autumn	63	Spring
56	60	25	7	53	Autumn	24	Spring
59	55	45	14	53	Autumn	43	Spring
64	63	12	11	53	Autumn	54	Spring
53	54	17	64	53	Autumn	63	Spring
13	7	26	20	54	Spring	23	Autumn
8	14	46	33	54	Spring	44	Autumn
63	64	11	12	54	Spring	53	Autumn
12	11	18	53	54	Spring	64	Autumn
41	31	40	3	63	Spring	23	Autumn
32	42	38	49	63	Spring	44	Autumn
54	53	64	17	63	Spring	53	Autumn
18	17	54	63	63	Spring	64	Autumn
24	44	37	41	64	Autumn	24	Spring
43	23	39	32	64	Autumn	43	Spring
11	12	53	18	64	Autumn	54	Spring
17	18	63	54	64	Autumn	63	Spring

Table 20: 32 Couples of opposite Wen hexagrams with their FuXi nuclear pairs

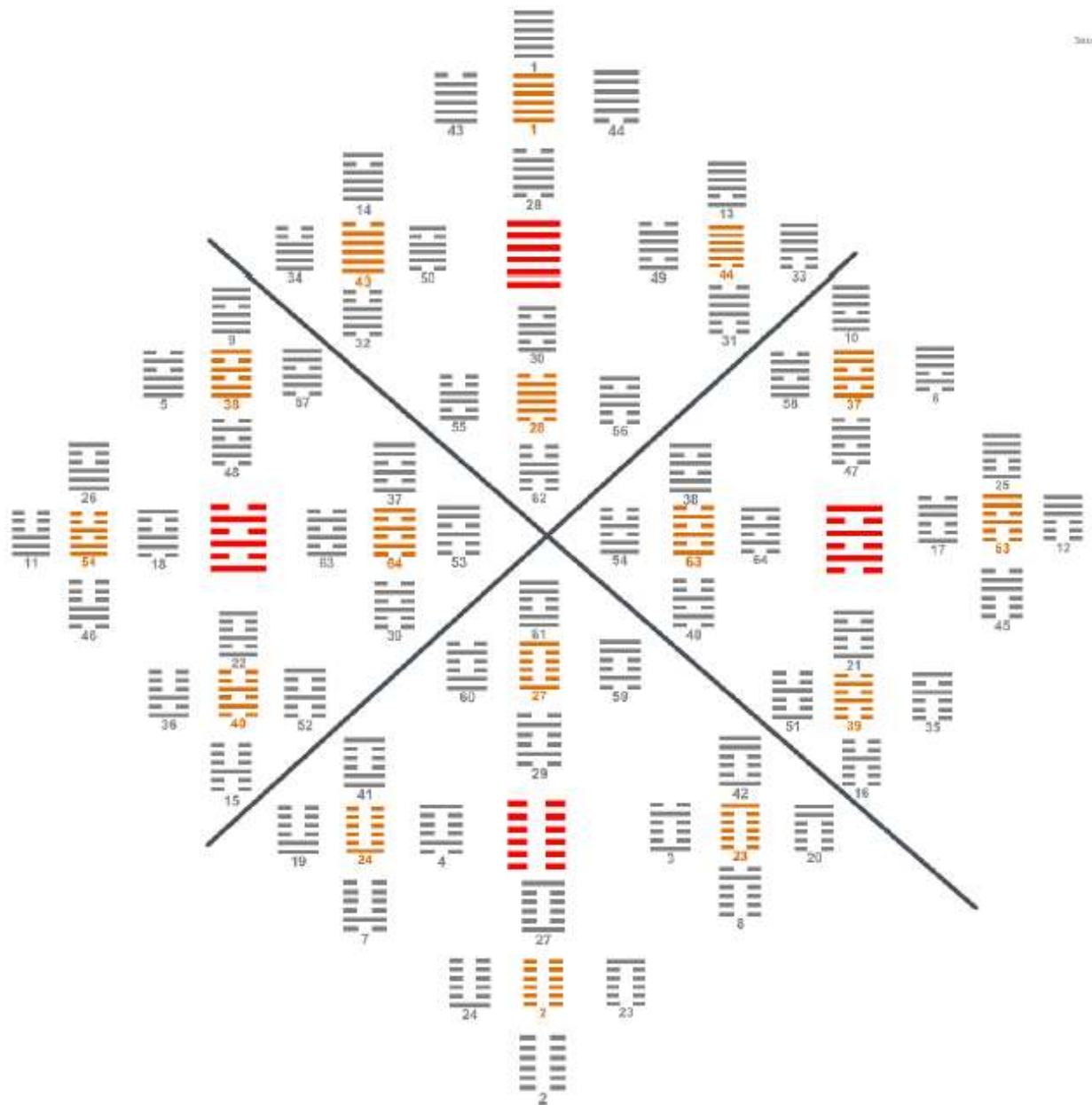## 9.7 The first and second degree nuclear families

Figure 19: The nuclear families[42] arranged according to the cardinal direction (South on the top, North at the bottom, East to the left, West to the right) The first degree nuclear are orange and the second degree nuclear red.

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[42] (Faure, 2006)